XVI. "Double Refraction and Dispersion in Iceland Spar. An Experimental Investigation with a Comparison with Huyghens' Construction for the Extraordinary Wave." By R. T. Glazebrook, M.A., Fellow of Trinity College, Cambridge. Communicated by J. Clerk Maxwell, M.A., F.R.S. Received June 12, 1879.

(Abstract.)

In the "Proceedings of the Royal Society," vol. xxvii, p. 498, the results of some experiments on double refraction in aragonite are given. From them I was led to the conclusion that there were slight differences between Fresnel's theory and experiment, and that possibly there was some connexion between these differences and the wave-length of the light used.

The present paper, of which this is an abstract, is an account of the results of similar experiments on Iceland spar. The method of the experiments is due to Professor Stokes ("Brit. Assoc. Reports," 1862).

A prism is cut from the crystal to be examined, and the position of its faces with reference to the crystallographic axes carefully determined. Then we may show with the usual notation that

$$\tan\frac{\phi'-\psi'}{2}=\tan\frac{i}{2}\cot\frac{\phi+\psi}{2}\tan\frac{\phi-\psi}{2}\quad .\ .\ .\quad \text{(I)}.$$

From this by observing the angles of incidence and deviation of a pencil of parallel rays incident on a prism of known refracting angle we can find $\frac{\phi'-\psi'}{2}$, and hence get ϕ' or ψ'; then from the formula

$$\frac{\text{V}}{v}=\frac{\sin\phi}{\sin\phi'}\quad .\ .\ .\ .\ .\ .\ .\ .\quad \text{(II)}.$$

V being the velocity of normal propagation in air, v in the crystal, we can find v in certain definite directions. But Huyghens' construction enables us to determine from theory the value of v, and we have hence a method of comparison.

Professor Stokes has made a series of experiments on Iceland spar by this method, and stated his results in "Proceedings of the Royal Society," vol. xx, 443. He found that to the limit of the error of his observations (about 0·0001 in the index) Huyghens' construction was fully confirmed.

The details of his experiments have not been published, and it ap-

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peared it might be useful to have a series of results which would serve in future as a test of any proposed theory of double refraction without the necessity of a further recourse to experiment. It is therefore largely with this object in view that I lay the results of my work before the Society.

I have sought to get a series of values of μ or $\frac{1}{v}$ for the three hydrogen lines C, F, g, in directions extending over an arc of 90° from the optic axis.

For this purpose I used four prisms, the edge of each was very approximately perpendicular to the optic axis.

The axis was inclined at angles of $-32°$, $14°$, $38°$, and $64°$ respectively, to the normal to one of the faces of the prism, the angles being considered positive when the optic axis and edge of the prism lie on the same side of the normal. By this means I was enabled to obtain a series of measurements in directions extending from about 5° on one side of the axis to 100° on the other. Each of the measured quantities used finally was the mean of two. The difference between the two was in only 18 per cent. of the observations as great as $20''$, so that the difference between the mean value taken and an extreme observation was only rarely as great as $10''$.

The spectrometer used was one by Grubb, of Dublin, kindly lent me by Professor Stokes. It reads with verniers to $10''$. I found that errors of $10''$ in the observed quantities taken so as to produce the greatest effect in the result would produce at most variations of ·00005 in the value of μ.

Precautions were taken to maintain the room at as nearly as possible a uniform temperature.

The position of the faces of the prism with reference to the rhombic faces of the crystal was determined by careful measurement. Each of the results adopted in the calculations was the mean of ten.

The results of the experiments are given in a series of tables.

For the theoretical calculation we have, if μ_1, μ_2, be the principal refractive indices, θ the angle between any wave normal and the optic axis,

$$\frac{1}{\mu^2}=\frac{\cos^2\theta}{\mu_1^2}+\frac{\sin^2\theta}{\mu_2^2}.$$

We require then to find μ_1 and μ_2.

μ_1 is the refractive index for the ordinary ray, and was found from observations on it in different positions of the prisms. μ_2 is the value of μ for a wave normal at right angles to the axis, and was found from observation on light travelling nearly in that direction.

The results are given below.

Values of principal refractive indices in Iceland spar for the three hydrogen lines.

	C.	F.	g.
μ_1	1·65436	1·66779	1·67553
μ_2	1·48456	1·49074	1·49430

The values of θ are determined from the known position of the wave in question, with reference to the normals to the faces of the prism, and of these normals with reference to the optic axis.

The results of the comparison are as follows :—

For the red line C, just in the neighbourhood of the major axis of the elliptic section, the curve given by experiment lies inside that given by theory, the two coincide at about 15° from this axis, henceforth onwards the experimental curve lies outside, and the two coincide again near the minor axis.

The greatest difference amounts to rather over ·0001, about half way between the axes. If we decrease slightly the major axis of the ellipse as given by theory, we may say that the result of experiment is expressed by an oval curve coinciding with Huyghens' ellipse at the extremities of its axes, and lying outside of it for the rest of its course, the greatest difference between the radii vectores being about midway between the axes, and there amounting to about $\frac{1}{10000}$th part of either.

The experiments for the ray F lead to similar results, the differences, however, between theory and experiment being markedly greater, and with the assumed values for μ_1, μ_2, negative near the minor axis, so that the experimental curve is on the inside there.

For g, taking the values of μ_1, μ_2, given above, the differences are negative, and the experimental curve o, the interior, near the axes, positive in the middle part of the arc, and the experimental curve o, lies outside, they amount to rather over ·0002 when a maximum.

Thus, the results for each of the three rays point to the same conclusions, and would lead us to infer that a section through the axis of the surface of wave slowness in Iceland spar would not be an ellipse, but an oval curve, lying slightly outside the ellipse, and that the difference between the two becomes greater as the wave-length decreases.

These differences, however, are very small, amounting to only about ·0002 for the violet ray.

I have, moreover, been able to prove that no variations in the values of the constants of theory or the position of the plane of the prism, with reference to the optic axis, will produce the effect required to bring theory and experiment into agreement. The results arising from a want of achromatism in the lenses have been considered at some length.

I have also endeavoured to show that, between the limits of the lines C and g, the relation between the refractive index and the wave-length may be expressed by means of the formulæ

$$\mu=\rho+\frac{\rho_1}{\lambda^2}+\frac{\rho_2}{\lambda^4}+ \quad . \; . \; .$$

Taking the three hydrogen rays, travelling in any the same direction, we have three equations, neglecting $\frac{\rho_2}{\lambda^4}$, &c.,

$$\mu_1=\rho+\frac{\rho_1}{\lambda_1^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1),$$

$$\mu_2=\rho+\frac{\rho_1}{\lambda_2^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2),$$

$$\mu_3=\rho+\frac{\rho_1}{\lambda_3^3} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (3).$$

The values of ρ found by combining these equations, two and two, differ on the average by about ·00006, and these differences are sometimes positive, sometimes negative.

Taking the mean of the values of ρ and the value of ρ_1 deduced from equation 1, I find the values deduced for μ_2 and μ_3 rarely differ from the observed values by as much as ·00006, the mean only being ·000032. This is considerably less in amount than the differences found by Dr. Hopkinson ("Proceedings of the Royal Society," vol. xxvi, p. 292) between the values of μ, as given by experiment on four kinds of glass, and deduced from three terms of a similar formula,

$$\mu=a+\frac{b}{\lambda^2}+\frac{c}{\lambda^4}.$$

I had hoped that, in the case of Iceland spar, it might be found that the quantity ρ obeyed Huyghens' construction accurately.

This has proved not to be the case; the differences are, on the whole, greater than those for the line g, and of the same sign.